Journal of X-Ray Science and Technology 11 (2003) 193–204
IOS Press

Modified fast marching and level set method for medical image segmentation

Fuping Zhu and Jie Tian*
Medical Image Processing Group, Institute of Automation, Chinese Academy of Sciences, Beijing P.O. Box 2728, 100080, China
Tel.: +86 10 62532105; Fax: +86 10 62527995

Abstract. In this paper, an interactive segmentation method that combines fast marching and level set method is proposed. Level set segmentation involves solving the energy-based active contour minimization problems by the computation of geodesics or minimal distance curves. First, by selecting the seed point, the fast marching method is used to extract rough boundaries of the interested object. We modified the traditional fast marching method to capture the weak edges by introducing watershed transform. Then, the contour obtained from the fast marching method mentioned above is regarded as an initialization and the level set method is used to finely tune the contour. The algorithm is demonstrated on some medical images: segmentation of knee tissues in CT image and segmentation of brain tissues in MR image. The results show that this method can remove the small regions obtained from fast marching method and converge the desired boundary.

1. Introduction

Segmentation is one of the bottlenecks of many image analysis and computer vision tasks ranging from medical image processing to robot navigation. Most of the further analysis, such as surgical planning, navigation, simulation, diagnosis and therapy evaluation relies on the results of the segmentation procedure. But image segmentation remains an open research problem and is a very difficult problem in practice. Due to the importance and difficulty of image segmentation, it has attracted great effort of many investigators since the seventies last century.

Currently, the numerous existing approaches of image segmentation may be classified into two main categories: boundary-based and region-based. In the boundary-based approaches, Edge detection is the earliest of the boundary-based methods, based on local gradients [1,19]. Threshold segmentation [20] is another common method. Active contours [2] have been introduced for segmenting deformable objects, by defining the internal and external functional and minimizing the functional whose local maximum is located at the object boundary. But, active contours are relatively noise sensitive, and their results depend on the initialization and they are not sufficiently topologically adaptive. In the region-based approaches, techniques, such as seeded region growing [3] or split-and merge were firstly introduced. Region growing methods generally require the desired regions to be homogeneous with respect to certain specified features in advance. Research on region-based segmentation methods has focused on either the design of feature measures and growing criteria or algorithm efficiency and accuracy.

*Corresponding author. E-mail: tian@doctor.com.

Level set method, developed by Osher and Sethian [4], based on the theory and numeric of weak solutions to surface propagation, has been used in the formulation of several region or boundary based approaches for image segmentation and offers highly robust and accurate techniques for tracking interfaces moving under complex motions. Level set segmentation involves solving the energy-based active contour minimization problems by the computation of geodesics or minimal distance curves [5–7,14]. The main idea of level set method is to represent a curve as the zero level set of a higher dimensional surface [8]. The entire surface is evolved to minimize a metric defined by the curvature and image gradient.

To segment an object from a medical image, in this paper, we use the idea of the fast marching and level set technique. First, the rough boundaries of interested object are extracted by fast marching method. The extracted edges are regarded as an initialization of level set method. We modified the traditional fast marching method in order to escape the error results. Then, the level set method is used to finely tune and smooth the contour acquired by fast marching method. The organization of this paper is as follows. Section 2 briefly introduces basic level set theory. Section 3 presents modified fast marching method. Section 4 gives the fine segmentation by level set method. We present some results using the new segmentation method in Section 5, followed by the conclusion and discussion in Section 6.

2. Basic level set theory

The main idea of level set method is to represent a closed curve $\Gamma(t)$ on the plane as the zero level set of a higher dimensional function Φ. The motion of the curve is then embedded within the motion of the higher dimensional surface. Let $\Gamma(t)$ be the closed interface or front propagating along its normal direction. This closed interface $\Gamma(t)$ can either be a curve in 2-D space or a surface in 3-D space.

Let $\Phi(x, t = 0)$, where $\in R^N$ be defined by $\Phi(x, t = 0) = d$, where d is the signed distance from position x to $\Gamma(0)$ and the plus (minus) sign is chosen if the point x is outside (inside) the initial front $4\Gamma(0)$ [9]. An Eulerian formulation is produced for the motion of this surface propagating along its normal direction with speed F, where F can be a function of the surface characteristics (such as the curvature, normal direction etc.) and the image characteristics (e.g. the gray level, gradient etc.). The equation of the evolution of Φ, inside which our surface is embedded as the zero level set is then given by the following equation:

$$\Phi_t + F|\nabla\Phi| = 0 \tag{1}$$

The major advantages of using this method over other active contour strategies include the following [10]. First, the evolving level set function $\Phi(x, t)$ remains a function, but the propagating front $\Gamma(t)$ may change topology, break, merge and form sharp corners as Φ evolves. Second, the intrinsic geometric properties of the front may be easily determined from Φ. For example, at any point of the front, the normal vector is given by $n = \nabla\Phi$. Finally, the formulation is unchanged for propagating interfaces in three dimensions.

One of the most popular level set algorithms is the so-called fast marching method. Now consider the special case of a surface moving with speed $F > 0$ (the case where F is everywhere negative is also allowed). We then have a monotonically advancing front whose level set equation is of the following form:

$$|\nabla \boldsymbol{T}|\boldsymbol{F} = 1 \tag{2}$$

This simply says that the gradient of arrival time is inversely proportional to the speed of the surface. There are two ways of approximating the position of the moving surface: iteration towards the solution by numerically approximating the derivatives in Eq. (1) or explicit construction of the solution function T from Eq. (2). Fast marching method depends on the latter approach.

Equation (2) is one form of the Eikonal equations. Sethian proved that it is equivalence to solve the following quadratic equation in order to get the arrival time T of the Eq. (2). The detailed approximating scheme refers to [8,11,12].

$$\left[\max(D_{ij}^{-x}T,0)^2+\min(D_{ij}^{+x}T,0)^2+\max(D_{ij}^{-y}T,0)^2+\min(D_{ij}^{+y}T,0)^2\right]^{1/2}=1/F_{i,j} \tag{3}$$

Where D^+ and D^- are backward and forward difference operators:

$$\begin{cases} \boldsymbol{D}^{+x}\boldsymbol{T}=\frac{T(x,y+1)-T(x,y)}{2} \& \boldsymbol{D}^{-x}\boldsymbol{T}=\frac{T(x,y)-T(x,y-1)}{2} \\ \boldsymbol{D}^{+y}\boldsymbol{T}=\frac{T(x+1,y)-T(x,y)}{2} \& \boldsymbol{D}^{-y}\boldsymbol{T}=\frac{T(x,y)-T(x-1,y)}{2} \end{cases} \tag{4}$$

The steps of the traditional fast marching method are as follows:

1. Initializing step:

   1) *Alive points:* Let A be the set of all grid points $\{i_A, j_A\}$ which represents the initial curve. In our algorithm, Alive points are the seeded points users assign to. See Fig. 1;
   2) *Narrowband points:* Let Narrowband points be the set of all grid point neighbors of A. In our algorithm, those are the 4-nearest points of the seeded points. Set $T(x,y)=1/F(x,y)$. See Fig. 1;
   3) *Faraway points:* Let Faraway points be the set of all others grid points $\{x,y\}$. Set $T(x,y)=$ TIME_MAX, See Fig. 1;

2. Marching forwards:

   1) Begin loop: Let $(i_{\min}, j_{\min})$ be the point in Narrowband with the smallest value for T;
   2) Add the point $(i_{\min}, j_{\min})$ to A; remove it from Narrowband;
   3) Tag as neighbors any points $(i_{\min}-1, j_{\min})$, $(i_{\min}+1, j_{\min})$, $(i_{\min}, j_{\min}-1)$, $(i_{\min}, j_{\min}+1)$ that are either in Narrowband or Faraway. If the neighbor is in Faraway, remove it from that list and add it to the set Narrowband;
   4) Recompute the values of T at all neighbors according to Eq. (3), selecting the largest possible solution to the quadratic equation;
   5) Return to top of Loop.

3. Modified fast marching method

From the steps of traditional fast marching method mentioned above, we know that all the pixels should be visited under the worst condition. During the course of fast marching method, how many points are visited on earth? This depends on the definition of the speed function and the selection of parameter value. In [12], R.Malladi defined the following speed function:

$$\boldsymbol{F}(x,y)=\boldsymbol{e}^{-\alpha|\nabla \boldsymbol{G}_\sigma * I(x,y)|} \tag{5}$$

Where $I(x,y)$ is the original image, ∇ is the gradient operator, $G_\sigma * I(x,y)$ is a Gaussian smooth operator for the original image, $0<\alpha<1$ is a weighting constants.

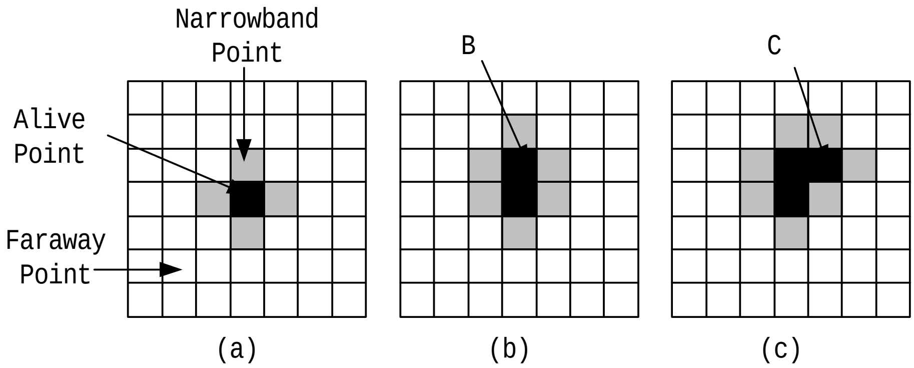

Fig. 1. Alive, Narrowband and Faraway points Fig. 1 (a) is the initialization procedure of fast marching method. The black grid point is the alive point. The gray grid points are narrowband points, and the white grid points are faraway points. Figures 1 (b) and 1 (c) is the marching procedure of fast marching method. In Fig. 1 (b) and (c), B and C are regarded as another two alive points.

Fig. 2. Segmentation results of traditional fast marching method cross the blurred edge Fig. 2(a) is an original image of CT knee. Those places that the green arrows point to are the seeded point and the blurred edge; Figure 2(b) is the segmentation results that cross the blurred edge.

From Eq. (5), we can know that the speed function $F(x, y)$ only depends on the edge information of the image, that is the gradient information. Because the speed function does not make full use of the global information of the image, for the blurred edge in the image, it is difficult to get good segmentation results. Figure 2 is an example.

Figure 2(a) is an original image of CT knee. Those places that the green arrows point to are the seeded point and the blurred edge; Figure 2(b) is the segmentation results that cross the blurred edge. Certainly this relies on the definition of the speed function and the selection of parameter value. The key of fast marching method is the definition of speed function. Due to the fact that speed function only depends on the gradient information (edge information), not the global information of the image region, it is easy to make mistakes in segmenting the blurred image boundary.

In general, it is well known that the image is composed of many small regions. Each region is of homogeneity, such as the contiguous intensity value, the similar texture structure. It is crucial for

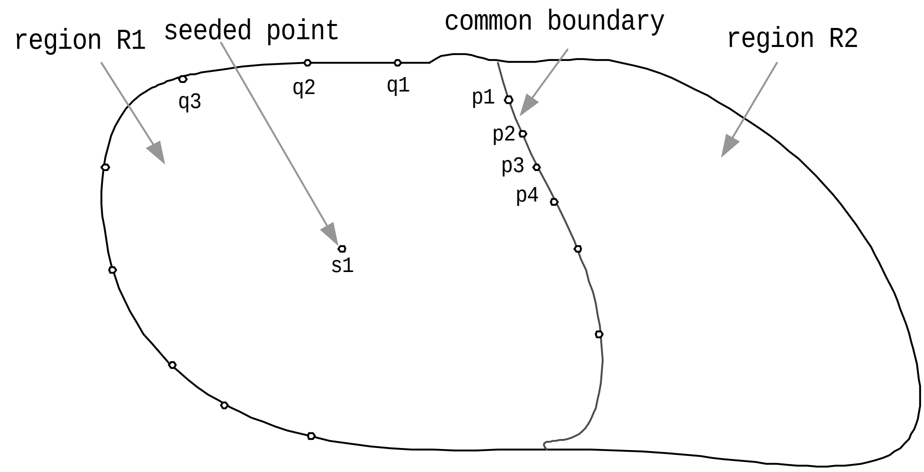

Fig. 3. The nearby regions R_1 and R_2 of Watershed transformation.

the final segmentation result to make full use of the region information. So, we introduce watershed transform [13] to over-segment the original image into many small regions.

The merit of introducing watershed transform lies in three aspects. Firstly, for fast marching method, let the seeded point be in the interior of the small region, we only need to calculate the arrival time of the seeded point to the boundary pixels. For the other pixels of the interior of the small region, due to the homogeneity with the seeded point, it is not necessary to calculate the arrival time of the seeded point to these pixels. So the algorithm's speed improves greatly. Secondly, the segmentation accuracy can be improved since the final segmentation results are bounded to be potential boundaries of objects. Finally, the statistical similarity degree of the nearby regions is a good reference of speed function of fast marching method.

Let the over-segmentation regions be $R_1, R_2, R_3, \ldots, R_n$, n is the number of the over-segmentation region. Let the statistical feature of the region R_i be: $\{SF_{i,j} | 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\}$. Where m is the number of features in the set, $SF_{i,1}$ is the mean gray value of the region and $SF_{i,2}$ is the variance of the region. The number of features used can be decided according to the character of the image. For example, if some objects in the image have apparent textures, texture features can be included in the feature set of the region. In our experiment, only $SF_{i,1}$ and $SF_{i,2}$ are used.

Figure 3 shows the nearby regions R_1 and R_2, after the image is watershed transformed. The seeded point (Alive point) locates in the region R_1. The red line in Fig. 3 is the common boundary of the region R_1 and R_2. p_1, p_2 and p_3 are the pixels of the common boundary, while q_1, q_2 and q_3 are the pixels of the other boundary of the region R_1.

When the seeded point s_1 arrive at the common boundary of the region R_1 and R_2, we can calculate the statistical features $SF_{1,j}$ and $SF_{2,j}$ of the region R_1 and R_2, and calculate the similarity degree between the region R_1 and R_2 as the following equation:

$$\boldsymbol{Sm}_{i,j} = \frac{1}{m} \textstyle\sum_{k=1}^{m} |\boldsymbol{SF}_{i,m} - \boldsymbol{SF}_{j,m}| \tag{6}$$

Where $Sm_{i,j}$ is the similarity degree between the region R_1 and R_2, m is the number of the statistical feature of the region R_i and R_j, $Sm_{i,m}$ is the statistical feature of the region R_i, $Sm_{j,m}$ is the statistical feature of the region R_j. We can define the speed function of the common boundary pixels of the region

R_i and R_j, after getting the similarity degree between the region R_1 and R_2:

$$\boldsymbol{F}_{i,j} = \boldsymbol{e}^{-\beta|\boldsymbol{Sm}_{i,j}|} \tag{7}$$

Where β is positive parameter. $(0 \leqslant \beta \leqslant 1)$

In summary, the steps of modified fast marching method are as follows:

1. Initialize step:
   1) *Alive points*: Let the seeded point and the interior pixels of the region where the seeded point locates be the alive points, $T(x, y) = 0$;
   2) *Narrowband points*: Let the boundary pixels of the region where the seeded point locate be the narrowband points, $T(x, y) = 1/F(x, y)$;
   3) *Faraway points*: Let the set of all others grid points $\{x, y\}$ be faraway points, $T(x, y) =$ TIME_MAX;
2. Marching forwards:
   1) Begin loop: Let (i, j) be the point in Narrowband with the smallest value for T, and the regions that the point (i, j) locates in be R_i and R_j;
   2) Mark the point (i, j) to alive point, remove it from Narrowband, calculate the similarity degree between the region R_i and R_j;
   3) Mark all of boundary points of the point (i, j) that are either in Narrowband or Faraway. If the boundary point is in Faraway, remove it from the Faraway and add it to the Narrowband;
   4) Recompute the values of T of all boundary points of the point (i, j) according to Eq. (3);
   5) Return to top of Loop.

4. Fine segmentation by level set method

The main characteristic of the level set method is its ability to pick up the right topology of the shape we are segmenting. The accuracy of the segmentation process depends upon where and when the propagating hypersurface needs to stop. For the fast marching method, the segmentation results rely on the definition of speed function to a greater degree. Whether the speed function adopts the definition of Eq. (5) or Eq. (7), there is a tunable parameter α or β which determines the value of speed function. It is important and also difficult to select the adaptive parameter value. So, on the condition of specified parameter value, it is necessary to use level set method to finely tune the rough contours obtained from fast marching method. In addition, through fast marching method, we can get the rough front and the location of each pixel. That is, we can determine where each pixel locates. It is useful and convenient to calculate the signed distance of the following level set method which is from each pixel to the front boundary.

The application of level sets in medical segmentation of medical imagery becomes extremely popular because of its ability to capture the topology of shapes in medical imagery. Since the proposal of level set method, many researchers [15–18] have succeeded in applying level set method to image processing and computer vision. The motion equation of level set method is given by Eq. (1). Now we can discretize it by finite difference approximation on a regular grid.

At first, from time and space we discretize the motion equation as follows:

$$\frac{\partial \Phi}{\partial t} \approx \frac{\Phi_{i,j}^{t+1} - \Phi_{i,j}^{t}}{\Delta t} \tag{8}$$

$$(|\nabla\Phi|)^2 \approx \left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial y}\right) \approx \left(\frac{\Phi^t_{i+1,j} - \Phi^t_{i-1,j}}{2\Delta x}\right)^2 + \left(\frac{\Phi^t_{i,j+1} - \Phi^t_{i,j-1}}{2\Delta y}\right)^2 \tag{9}$$

Where i and j are the discrete coordinates of each pixel. So, the Eq. (1) is turned into the following equation:

$$\Phi^{t+1}_{i,j} = \Phi^t_{i,j} + F(K)\left|\left(\frac{\Phi^t_{t+1,j} - \Phi^t_{i-1,j}}{2\Delta x}\right)^2 + \left(\frac{\Phi^t_{i,j+1} - \Phi^t_{i,j-1}}{2\Delta y}\right)^2\right|\Delta t \tag{10}$$

Where $\boldsymbol{F}$ is a scalar speed function. General speaking, the speed function $\boldsymbol{F}$ depends on the following three terms:

1) Local properties of the front, such as the local curvature;
2) External parameters related to the input data (image gradient for instance);
3) Additional propagation terms.

Here, we adopt the speed function defined in [12] to tune the rough contour came from fast marching method. It includes two terms. The first one is to use a regularization term to uniformly penalize and smooth regions of high curvature. The second term is the driving force that molds the initial surface into desired anatomical shapes. Specifically, the equation of motion is as follows:

$$\Phi_t + \boldsymbol{k}_I(1 - \varepsilon H)|\nabla\Phi| - \beta\nabla P \cdot \nabla\Phi = 0 \tag{11}$$

Where,

$$\boldsymbol{k}_I = \frac{1}{1 + |\nabla \boldsymbol{G}_\sigma * I(x,y)|} \tag{12}$$

The second term denotes the projection of a force vector on the surface normal. This force is regarded as the gradient of an external field:

$$P(x,y) = -|\nabla \boldsymbol{G}_\sigma * I(x,y)| \tag{13}$$

Where coefficient $\beta > 0$, controls the strength of this attraction. $\varepsilon > 0$. Where $\boldsymbol{H}$ is the mean curvature given by the following equation:

$$H = \frac{\Phi_{xx}\Phi_y^2 + \Phi_{yy}\Phi_x^2 - 2\Phi_{xy}\Phi_x\Phi_y}{(\Phi_x^2 + \Phi_y^2)^{3/2}} \tag{14}$$

In summary, the overall steps to segment a medical imagery are as follows:

1) Perform basic image filtering and enhancement, such as applying anisotropic diffusion filtering to the original image to eliminate the noise and enhance the edge;
2) Obtain an over-segmentation of medical imagery by watershed method;
3) Get the rough contour by modified fast marching method;
4) Use level set method defined in Eq. (11) to tune initial boundary.

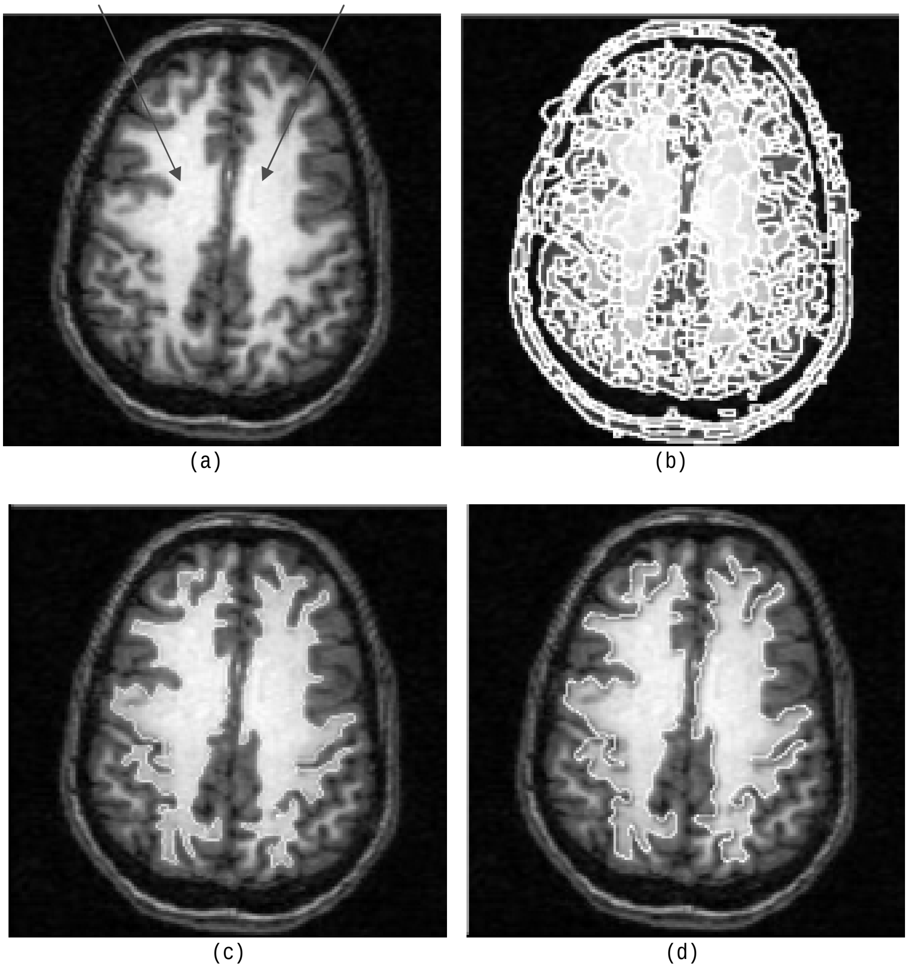

Fig. 4. Segmentation of the knee bone tissue Fig. 4 (a) is a MR imagery of a brain, the seed points are selected in the left and right middle of white matter region (indicated by the arrow); (b) over-segmentation by watershed transform; (c) rough segmented results by modified fast marching method; (d) fine segmented results by level set method.

5. Experiment results

This method was used in the medical image processing system (3DMED: http://www.3dmed.net/) developed by our lab. It has successfully segmented scores of CT images and MR images. Experimental results show that the proposed method can produce better segmentation results. Some representative images are shown in Figs 4, 5, 6 and 7.

Figure 4 shows the segmented results of brain white matter performed on the MR imagery. The first one is the original MR image, and the seed points are indicated by the arrows. The second one is the over-segmented result by the watershed transform. Figure 4 (c) and (d) present the rough and fine results by the fast marching and level set method separately. Compared to the results of fast marching method, the proposed segmentation method is able to capture the more smoothed boundary. Figure 5 presents segmented result of CT knee. From Fig. 5 (c), we can see that the modified fast marching method converge the blurred edges corresponding to Fig. 2. But at the same time, it produces some small

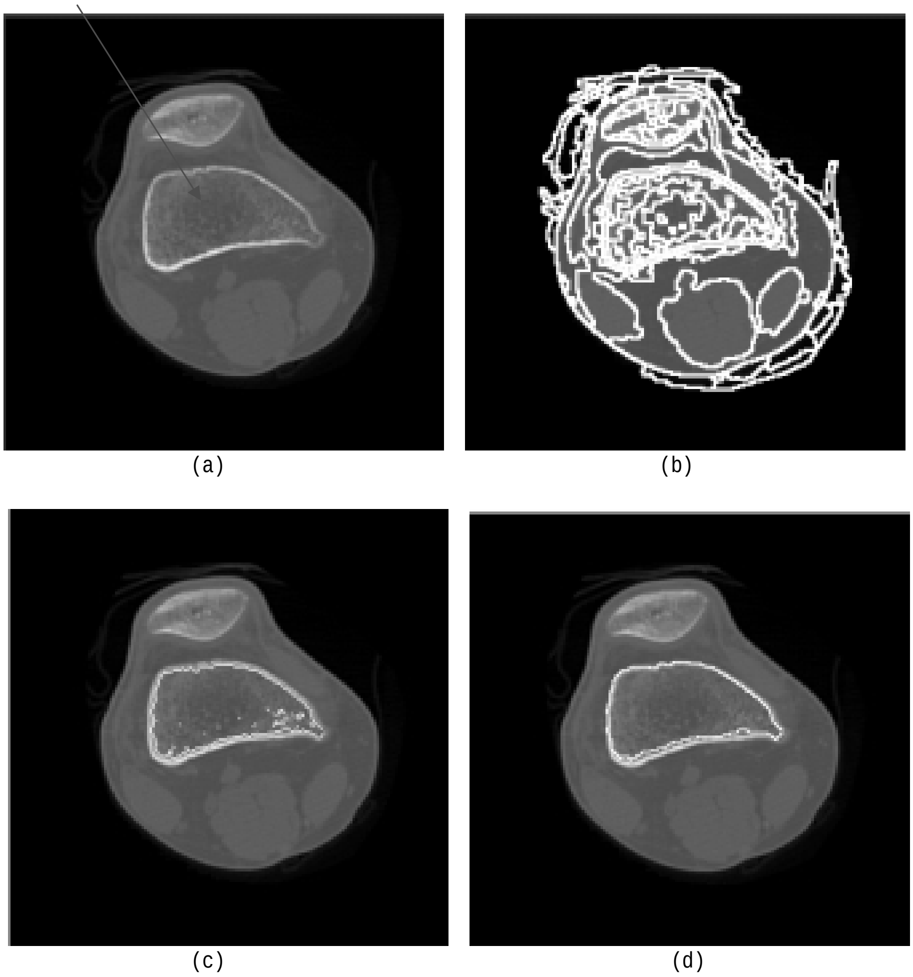

Fig. 5. Segmentation of the knee fat tissue Fig. 5 (a) is a CT imagery of a patient's knee, the seed point is selected in the middle of knee fat region (indicated by the arrow); (b) over-segmentation by watershed transform; (c) rough segmented results by modified fast marching method; (d) fine segmented results by level set method.

regions. Figure 5 (d) not only gives a smoothed contour, but also merges some small regions. Figure 6 gives the same conclusion as Fig. 5.

Figure 7 gives segmented results of brain tumor from Harvard Medical School who provided Brain Tumor Segmentation Database. All images are scanned with a slice thickness of 1.5 mm. The MR datasets consist of 256×256 matrix, with 124 slices. The voxel size is $0.9375 \times 0.9375.5$ mm in all cases. Figure 7 (a)–(d) are same as Figs 5 and 6. We compare our segmented result to expert manual result. See Fig. 7 (e) and (f).

6. Conclusion

In this paper, we proposed a practical two-stage method integrating fast marching with level set method, and applied it to segment medical images. First, fast marching method is used to extract the

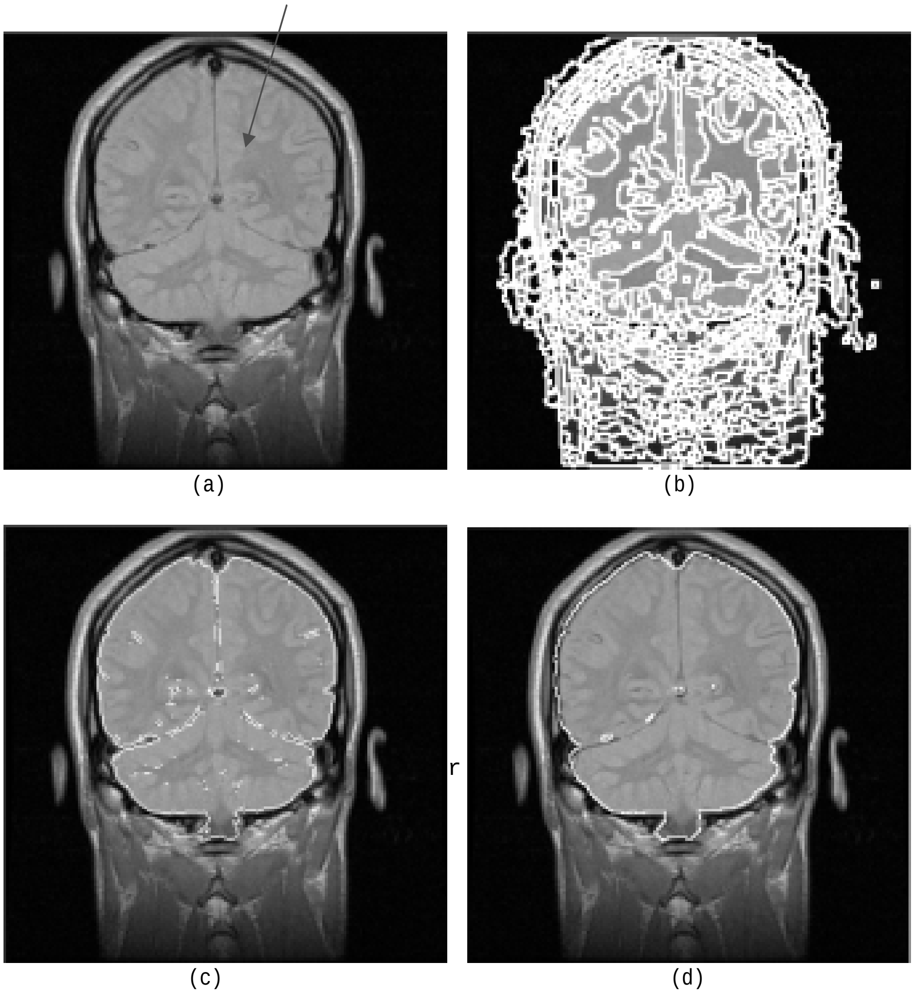

Fig. 6. Segmentation of the brain tissue Fig. 6 (a) is a MR imagery of a brain from the ISBR (http://neuro-www.mgh. harvard.edu/cma/ibsr); the seed point is selected in the middle of brain tissue region (indicated by the arrow); (b) over-segmentation by watershed transform; (c) rough segmented results by modified fast marching method; (d) fine segmented results by level set method.

rough contours. Then level set method is utilized to finely tune the initial boundary. Moreover, Traditional fast marching method was modified by the use of watershed transform. Experiment results show that the method is feasible in medical imaging and deserves further research. It could be used to segment the white matter, brain tumor and other small and simple structured organs in CT and MR images. In the future, we will integrate level set method with statistical shape analysis to make it applicable to more kinds of medical images and have better robustness to noise.

Acknowledgments

We are grateful to the editors, anonymous reviewers for their careful reading and helpful suggestions. Thanks for the Center for Morphometrics Analysis at Massachusetts General Hospital who provided

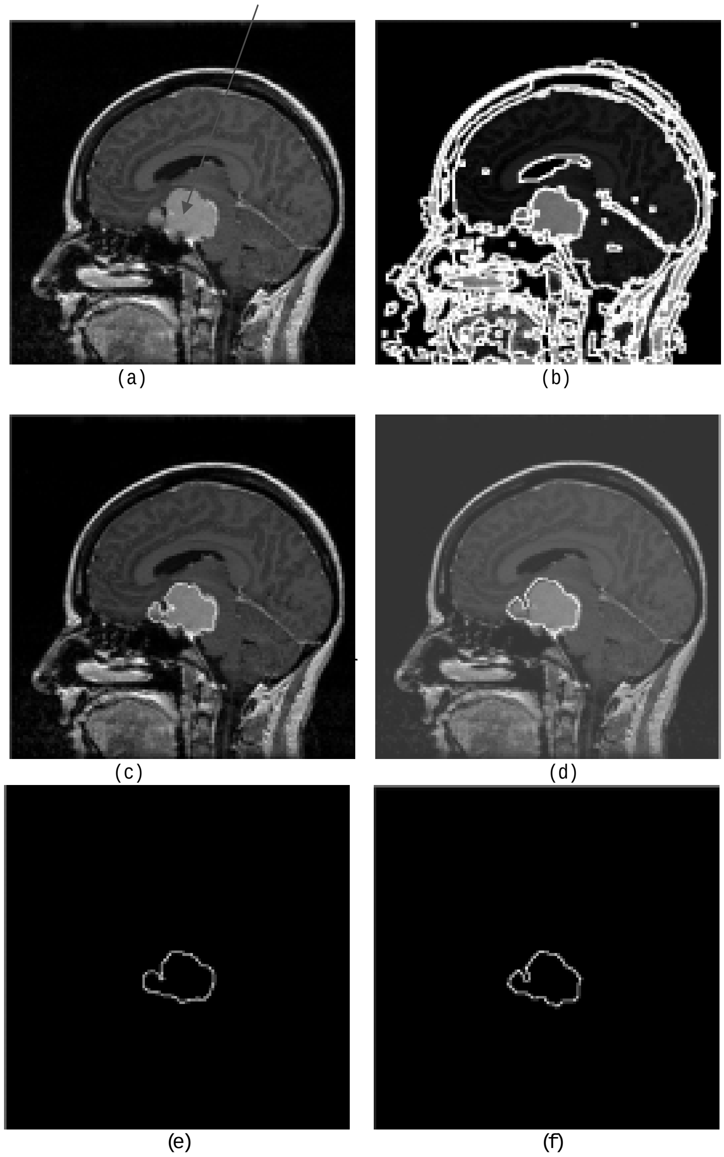

Fig. 7. Segmentation of the brain tumor Fig. 7 (a) is a MR brain imagery from the Brain Tumor Segmentation Database in Harvard Medical School (tumorbase@bwh.harvard.edu); the seed point is selected in the middle of brain tumor region (indicated by the arrow); (b) over-segmentation by watershed transform; (c) rough segmented results by modified fast marching method; (d) fine segmented results by level set method; (e) expert manual result; (f) our method result.

the IBSR database and Harvard Medical School who provided Brain Tumor Segmentation Database. This paper is supported by the National Science Fund for Distinguished Young Scholars of China under Grant No. 60225008, the Special Project of National Grand Fundamental Research 973 Program of China under Grant No. 2002CCA03900, the National High Technology Development Program of China under Grant No. 2002AA234051, the National Natural Science Foundation of China under Grant Nos. 60172057, 69931010, 60071002, 30270403, 60072007.

References

[1] Peihua Qiu, Some recent developments on edge detection and image reconstruction based on local smoothing and nonparametric regression, In book *recent research developments in pattern recognition* **1** (2000), 41–49.

[2] T. McInerney, D. Terzopoulos, Deformable models in medical image analysis: a survey, *Medical Image Analysis* **1**(2) (1996), 91–108.

[3] Rolf Adams, Leanne Bischof, Seeded region growing, *IEEE Transactions on Pattern Analysis and Machine Intelligence* **16**(6) (June, 1994), 641–647.

[4] S. Osher and J. Sethian, Fronts propagating with curvature dependent speed: Algorithms based on the Hamilton-Jacobi formulation, *Journal of Computational Physics* **79** (1988), 12–49.

[5] V. Caselles, R. Kimmel and G. Sapiro, Geodesic active contours, *International Journal of computer Vision* **22**(1) (1997), 61–79.

[6] A. Kichenassamy, A. Kumar, P. Olver, A. Tannenbaum and A. Yezzi, *Gradient flows and geometric active contour models*, in IEEE International Conference Computer Vision, 1995, pp. 810–815.

[7] R. Malladi, J. Sethian and B. Vemuri, Shape modeling with front propagation: A level set approach, *IEEE Transactions on Pattern Analysis and Machine Intelligence* **17**(2) (February, 1995), 158–175.

[8] J.A. Sethian, *Level Set Methods*, Cambridge University Press, 1996.

[9] J. Suri, K. Liu, S. Singh, S. Laxminarayana and L. Reden, Shape Recovery Algorithms Using Level Sets in 2-D and 3-D Medical Imagery A State-of-the-Art Review, IEEE Transactions in Information Technology in Biomedicine (ITB), 2001.

[10] X. Zeng, L.H. Staib, R.T. Schultz and J.S. Duncan, Segmentation and Measurement of the Cortex from 3D MR Images, In Proceedings of the First International Conference on Medical Image Computing and Computer-Assisted Intervention, October, 1998.

[11] J.A. Sethian, A fast marching level set method for monotonically advancing fronts, *In Proceedings of the National Academy of Sciences*, USA **93** (1996), 1591–1595.

[12] R. Malladi and J.A. Sethian, An O(N log(N)) algorithm for shape modeling, *In Proceedings of National Academy of Sciences*, USA **93** (Sept. 1996), 9389–9392.

[13] L. Vincent, P. Soille, Watersheds in digital spaces: an efficient algorithm based on immersion simulations, *IEEE Transactions on Pattern Analysis and Machine Intelligence* **13**(6) (1991), 583–589.

[14] Michael E. Leventon, W. Eric and L. Grimson, Olivier Faugeras, William M. Wells Iii, Level set based segmentation with intensity and curvature priors, *IEEE Workshop on Mathematical Methods in Biomedical Image Analysis* (*MMBIA'00*), Hilton Head, South Carolina, June 11–12, 2000.

[15] N. Paragios, R. Deriche, Geodesic active contours and level sets for the detection and tracking of moving objects, *IEEE Transactions on Pattern Analysis and Machine Intelligence* **22**(3) (2000), 266–280.

[16] C. Samon, L. Blanc-Feraud, G. Aubert, Z. Josiane, Level set model for image classification, *International Journal of computer Vision* **40**(3) (2000), 187–197.

[17] K. Museth, D. Breen, L. Zhukov and R. Whitaker, Level Set Segmentation From Multiple Non-uniform Volume Datasets, Proceedings of IEEE Visualization 2002, October 2002, pp. 179–186.

[18] Tony F. Chan, Luminita A. Vese, Active contours without edges, *IEEE Transaction on image processing* **10**(2) (FEB., 2001), 266–277.

[19] Yao Lin, Jie Tian, Huiguang He, Image Segmentation via Fuzzy object extraction and Edge Detection and its Medical Application, *Journal of X-ray Science and Technology* **10** (Numbers 1–2/2001), 95–106.

[20] Xingfei Ge, Jie Tian, Fuping Zhu, Segmentation of medical images based on homogram thresholding, Proceedings of SPIE Medical Imaging 2003, San Diego, USA, February, 2003.